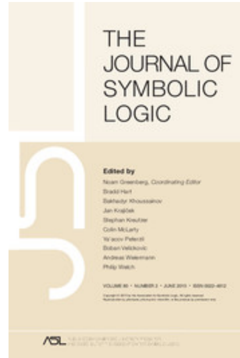

Brady Allen H.. The busy beaver game and the meaning of life. *The universal Turing machine, A half-century survey*, edited by Herken Rolf, Kammerer & Unverzagt, Hamburg and Berlin, and Oxford University Press, Oxford and New York, 1988. pp. 259–277.

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The Journal of Symbolic Logic / Volume 56 / Issue 03 / September 1991, pp 1091 - 1091
DOI: 10.2307/2275078, Published online: 12 March 2014

mathematics, the last one being in the form of a discussion between various imaginary characters, trying to get a balanced view of the successes, obstacles, and prospects in this area.

Accessible to the general reader, the article is a very good and up-to-date summary of work on the interplay between logic and computation. J. C. SHEPHERDSON

ALLEN H. BRADY. *The busy beaver game and the meaning of life.* Ibid., pp. 259–277.

In 1936–1937 Turing (II 42) presented a universal computing machine having twenty-eight states and fifteen symbols, with some sequences of moves and prints as atomic acts. Following Kleene (XIX 215) it is now usual to have one print and one move as constituting an atomic act (simply one print or one move, however, is also used as an atomic act). It is trivial that two symbols, 1 and 0 (blank, almost everywhere on the tape), and many states suffice; and it is known that two states and many symbols suffice (Shannon, XXXVI 532). The state-symbol product is used as a measure of complexity, and a seven-state by four-symbol universal Turing machine is known to exist (Minsky, XXXI 655).

The paper under review discusses Rado's Busy Beaver game (XXXII 524), Conway's game of Life, and similar games that yield some very fast growing functions. Some values or bounds for small arguments are given and it is discussed where the rapid growth starts. If the numbers represent some kind of *recursive strength* of the devices involved, one can guess where universal devices might exist.

The Busy Beaver game concerns n-state by two-symbol Turing machines starting on the empty tape and eventually stopping. $\Sigma(n)$ is the maximal score (i.e., number of 1 symbols left on the tape after stopping) for such machines and $S(n)$ is the maximal number of steps such a machine can run before it stops. $\Sigma(n)$ and $S(n)$ can easily be shown to grow faster than any recursive function.

Some values for Σ and S are known, for example, for $n = 2, n = 3$ (Lin and Rado, XL 617), and $\Sigma(4) = 13, S(4) = 107$ (Brady). The hard case is for $n = 5$. When computers became cheaper and faster many high-scoring and long-running (5×2)-machines were found, the best reported here (by Uhing) showing $\Sigma(5) \geqq 1{,}915$ and $S(5) > 2.3 \times 10^6$. The author points out that when looking at such tricky machines found by computer one can verify that they stop but one has virtually no idea why they stop. And since whatever bounds on space and time are fixed for a computer search, there are left in the search space immensely many machines that must be treated individually, it will probably never be possible to prove mathematically that they will not stop. Even if one does get the exact values for $n = 5$, one might never be able to prove it rigorously. The case $n = 6$ is quite intractable. (Recently Marxen and Buntrock have raised the $n = 5$ scores to $\Sigma(5) \geqq 4{,}098$ and $S(5) > 4.7 \times 10^7$; see ***Bulletin of the European Association for Theoretical Computer Science***, no. 40 (1990) pp. 247–251.)

The Busy Beaver game can be generalized by admitting more symbols, the functions Σ and S then becoming two-place functions. The author concentrates on the function S, which is a better measure of complexity. He gives some lower bounds for two states and three and four symbols and discusses the complexity of several cases.

Cellular automata have patterns of cells that are active devices working in parallel, each cell being influenced by its neighbors. The author discusses linear cellular automata introduced by Varshavsky, the well-known two-dimensional game of Life, and simulation by Turing machines operating on a two-dimensional tape. He also considers variants he calls TurNing machines which have a forward direction that can be turned. TurNing machines can also be defined in a natural way for triangular grids. Some examples are given and their expected complexity is discussed.

The paper contains several interesting philosophical remarks and closes with some conjectures and predictions, a few of which are given here: There is a universal one-state TurNing machine. The halting problem for three-state by two-symbol Turing machines is decidable. There exists a four-state by two-symbol universal Turing machine if halt entries are excluded. ARNOLD OBERSCHELP

UWE SCHÖNING. *Complexity theory and interaction.* Ibid., pp. 561–580.

Imagine a resource-bounded Turing machine (TM) augmented by the ability to interact with some outside agency. That is the theme of this survey paper, which has exceptionally clear exposition, gives accurate definitions, concentrates on major results, and provides many of the proofs.

The basic definitions are explained in terms understandable to any logician. Included are the major computational complexity classes, with focus on polynomial time, *P*, and its non-deterministic counterpart, *NP*. The phenomenon of *NP-completeness* is discussed, and an analogy with the arithmetical hierarchy is exploited to define the *polynomial time hierarchy*.